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Comparison of semi-classical and quantum theories for a single-mode laser†

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Abstract. The dependence on cavity Q of the intensity of a single-mode laser above threshold is examined for the quantum and semi-classical theories due to Scully and Lamb, and Stenholm and Lamb, respectively. The two theories predict different functional dependencies, but comparison with previously published results show that both theories are satisfied for the small relative excitations possible in a He–Ne laser.

Recently Sayers *et al.* (1969) have experimentally investigated certain predictions of Scully and Lamb's (1967) quantum theory of the laser. It was shown that away from threshold the theory predicts that the expected number of photons in a single mode can be written as

$$\langle n \rangle = \frac{\{(L_c + L_m)L' - 1\}L_c\langle n \rangle_p}{L_m} \qquad (1)$$

where L_c is the residual cavity loss, L_m the maximum inserted loss, L the instantaneous inserted loss and $L' = (L_c + L)^{-1}$. This expression was experimentally confirmed by use of the technique of single-pass gain measurement described by Allen *et al.* (1968), as were the predictions of Scully and Lamb concerning $\langle n \rangle$ near threshold.

It is thought worth while to point out that the same experimental results may be used to test a new semi-classical theory of the laser due to Stenholm and Lamb (to be published). In a previous classic paper Lamb (1964) developed a semi-classic theory, but the calculations were carried out only to third order in the electromagnetic field and consequently the region of validity was not expected to be large. Indeed it has been shown (Sayers *et al.* 1969) that for a single mode this theory only adequately describes the behaviour of the laser intensity as a function of cavity Q for values of the relative excitation parameter η in the range 1·02 to 1·10.

Stenholm and Lamb have shown that it is possible to develop a theory for arbitrarily large values of the electromagnetic field. The result of the calculation to relate laser intensity to the relative excitation intensity for a single mode is expressed as a continued fraction. The lowest approximation to the solution of the basic equations is found to give equations in exact agreement with the result obtained by Lamb (1964, § 18) where, in single-mode operation, the approximation is shown to be equivalent to a rate equation approach. Hence Stenholm and Lamb call it the rate equation approximation.

Numerical calculation of the results for the continued fraction approach and the results of the rate equation approximation were compared by Stenholm and Lamb in figures 15 and 16 of their paper. The rate equation approximation may be seen to follow the exact results closely, whereas the third-order theory deviates from it appreciably at $\eta \geqslant 1{\cdot}1$ in agreement with Sayers *et al.* It is clear that the results of the rate equation approximation deviate appreciably from the exact solution only for $\eta \geqslant 3{\cdot}5$ when the laser frequency is at the centre of the gain profile, and for a laser detuned by more than γ_{ab} from the line centre the rate equation approximation gives a good approximation even for the largest intensities considered.

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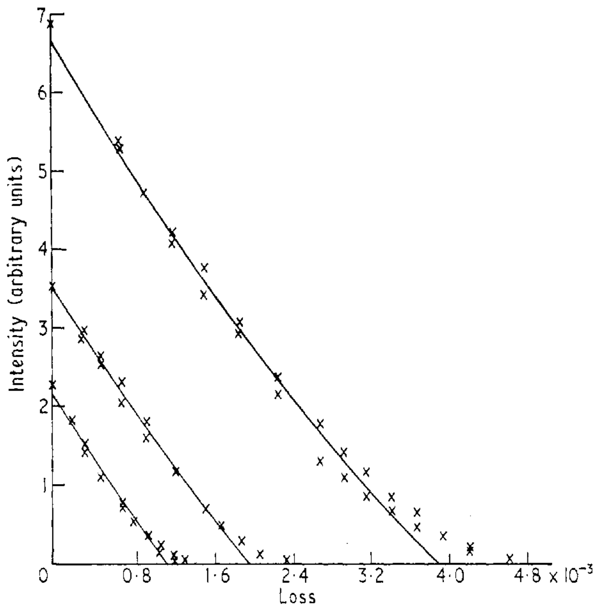

Figure 1. Comparison of experimental results with the theory of Stenholm and Lamb (full curve) for single-mode intensity as a function of inserted cavity loss, for three values of peak intensity I_p.

If the rate equation approximation solution for laser intensity is examined for a single mode at a fixed frequency, it may be shown to be in conflict with Scully and Lamb's equation in the way in which it depends upon certain variables. If Stenholm and Lamb's equations (98) and (100) are combined with (12) and the equation transcribed into the same measurables as before, then

$$\frac{I}{I_p} = \left\{\left(\frac{L_c+L_m}{L_c+L}\right)^2 - 1\right\}\left\{\left(\frac{L_c+L_m}{L_c}\right)^2 - 1\right\}^{-1}. \tag{2}$$

Using the same experimental results as before, it was found that away from threshold the dependence of I upon the losses described in equation (2) was satisfied as well as, or arguably slightly better than, the dependence predicted by the quantum theory (see figure 1). Understandably only the quantum theory explains the intensity very near threshold.

It should perhaps be noted that no very special precautions were taken to ensure that there was no change in oscillation frequency during the course of a measurement except that the observation time was kept very short. Also the range of intensities was not very large since a He–Ne laser was used. Since the rate equation approximation solution embraces the third-order perturbation theory it was to be expected that it would work well in the range $\eta = 1{\cdot}02$ to $1{\cdot}10$. Nevertheless, the present results show that the new theory works very well in the range 1·10–1·20 as well, and this confirms that the new theory extends the intensity range which has been well described semi-classically. Presumably at higher values of η the difference in the functional dependencies predicted by equations (1) and (2) will begin to show, but it will not be possible to do the necessary experiment with a He–Ne laser. It is expected, since the quantum theory of Scully and Lamb deals with the case of zero atomic motion, that the semi-classical theory will give the best agreement with experiment.

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References

Allen, L., Jones, D. G. C., and Sayers, M. D., 1968, *J. Phys.* E (*J. Sci. Instrum.*), [2], **1,** 133–5.
Lamb, W. E., Jr., 1964, *Phys. Rev.*, **134,** A1429–50.
Sayers, M. D., Allen, L., and Jones, D. G. C., 1969, *J. Phys.* A (*Gen. Phys.*), [2], **2,** 102–5.
Scully, M. O., and Lamb, W. E., Jr., 1967, *Phys. Rev.*, **159,** 208–26.